METHODOLOGICAL APPROACHES TO POVERTY, WELFARE, AND INCOME INEQUALITY COMPARISON IN ZIMBABWE: Lessons From the Study On Peru.*

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HARARE, ZIMBABWE
DECEMBER, 1995

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* Kaliyati, J., 'Welfare, Poverty, and Inequality in Peru: An Application of Recently Developed Evaluation Techniques to Economic Development', (University of Alabama, U.S.A., Ph.D. Dissertation, 1995).

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FIGURE | PAGE

ACRONYMS

FOD = FIRST ORDER DOMINANCE

FSD = FIRST DEGREE STOCHASTIC DOMINANCE

LSMS = LIVING STANDARDS MEASUREMENT SURVEY

LD = LORENZ DOMINANCE

PD = POVERTY DOMINANCE

PLSSs = PERU LIVING STANDARDS SURVEYS

SOD = SECOND ORDER DOMINANCE

SSD = SECOND DEGREE STOCHASTIC DOMINANCE

SWF = SOCIAL WELFARE FUNCTION

Methodological Approaches to Poverty, Welfare and Income Inequality Comparisons in Zimbabwe: Lessons from the study on Peru*

1. Introduction

Discussion on Poverty in Zimbabwe has focussed on the following areas:

(a) Poverty Profiles;
(b) Poverty Monitoring; and
(c) The Impact of ESAP on Poverty.

The debate has been extremely useful in the sense that it provided useful information on people's perceptions about poverty and how it can be dealt with. While poverty is almost common knowledge to most people, its measurement, let alone monitoring, is an intricate process whose outcome is highly sensitive to the measurement method used. This paper contributes to the debate on poverty in Zimbabwe by reviewing some of the methodologies used in the study of welfare, income inequality, and poverty in Peru (See Kaliyati,1995).

2. Poverty and Welfare

Poverty is part of a wider concept called welfare. Welfare itself being an ordinal concept of measuring an individual's or a nation's well-being. Welfare comparison within and across nations, regions or even between population sub-groups comprises four sets of related analyses. These are:

(a) Income inequality comparisons;
(b) Poverty comparisons;
(c) Overall welfare comparisons; and
(d) Income mobility analysis.

Income inequality comparisons are important in that if we have two countries with identical wealth, but in one of the countries the wealth is less equally distributed than in another, the country with a more equally distribute wealth is considered to have a higher level of welfare. The need for poverty comparisons is obvious. Less poverty

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increases overall welfare. Overall welfare comparisons involve chosing an appropriate welfare measure, make the necessary adjustments for time and household size and composition differences, and compare welfare between population sub-groups, regions, across countries or across time. Mobility matrix analysis involves tracing the same individuals or households across time to see whether or not their welfare/social rankings have changed. This is done using panel data. This paper gives the methodological approaches used in each of the four types of analyses mentioned above and demonstrates how the results are interpreted using the study on Peru (Kaliyati, 1995). The rest of the paper is as follows: Section 3 looks at overall welfare comparisons. Section 4 reviews the methodologies used in the measurement of welfare, income inequality, and poverty. Section 5 deals with issues related to the measurement of poverty. Section 6 reviews the statistical inference procedures used in the dominance methodology. Section 7 shows how income mobility analysis is carried out. Finally, section 8 concludes by relating the welfare comparison methodologies to the poverty debate in Zimbabwe.

3. Overall Welfare Comparisons

Welfare comparisons are aimed at answering the following research questions:

(a) How has welfare been changing across time?

(b) How does welfare compare between:
   - (i) income sub-groups;
   - (ii) different regions;
   - (iii) urban and rural areas;
   - (iv) male headed households and female headed households;

(c) How equally is income distributed and how did this distribution change across time?

(d) How does the distribution of income compare between:
   - (i) income sub-groups;
   - (ii) different regions;
   - (iii) urban and rural areas;
   - (iv) male headed households and female headed households;

(e) Has poverty been increasing or decreasing?

(f) How does poverty compare between:
   (i) different regions;
   (ii) urban and rural areas;
   (iii) male headed households and female headed households;

These comparisons are made using the stochastic dominance methodology which is discussed below.

Traditionally incomes have been used as a welfare indicator. In this approach higher incomes imply a higher level of welfare. This approach has since been overtaken by the use of expenditure as a welfare indicator. Two arguments are advanced in favour of the latter approach. The first is that individuals' or households' welfare is raised not by the income available for consumption, but by the goods and services they consume. This is particularly true for developing countries where the concept of extended families has resulted in substantial amounts of money being transferred between households. The second reason is that people, in general, are more reluctant to reveal their salaries than their consumption expenditures. Consumption expenditure data therefore tends to be more accurate compared to income data (see Ravallion, 1993; Louat, Grosh, and van der Gaag, 1993). An additional reason is that consumption expenditure is a better proxy for permanent income than absolute incomes. More recently foodshares have been used as a welfare indicator (Kaliyati, 1995; Bishop, Formby, and Zheng, 1995). The use of foodshares as a welfare indicator proved particularly useful than expenditure in the study of welfare, income inequality, and poverty in Peru (Kaliyati, 1995). This is because during periods of hyper-inflation, such as the study period in the Peruvian study, high price variability renders the CPI deflator highly unreliable that the results obtained from analyses based on deflated data using such CPI deflators are equally suspect. Foodshares, being proportions of food expenditure to total expenditure, are invariant to the method of deflation. In this paper only the methodology that uses household expenditure as a welfare indicator is considered.

Before expenditure data is used to make welfare comparison several adjustments need to be made on the original data. First, we need to adjust the data to take care of differences in family sizes and composition. When comparisons are across time, we also need to adjust the expenditure data to their constant price levels. The latter is fairly standard. The former is subject to a lot of debate. Several ways of accounting for differences in household sizes and composition have been suggested. The general methodology involves assigning different members of the household weights ranging from zero to one, and dividing the household expenditure by the sum of the weights (the sum of these weights is also referred to as the 'adult equivalence scale') to obtain household expenditure per adult equivalence. When all the members of a household are each assigned a weight of one then the expenditure per adult equivalence is simply the per capita expenditure. This equivalence scale does not take into account two things. First, it does not take into consideration the fact that the proportion of total expenditure consumed by an adult is different from that for children. Second, it does not take economies of scale into account. By economies of scale we mean that if two adults are staying together they would not require two TVs or two stoves, thus by living together their expenditure is bound to be less. Cutler and Katz (1992) suggested an adult equivalence scale which can be written as:

$AE = [A + 0.4C]^{¾}$, where

AE = Adult Equivalence,

A = Number of adults, and

C = Number of children under the age of eighteen.

This equivalence scale takes into account both differences in household composition and economies of scale. Glewwe and Hall (1994) suggested another equivalence scale which gives varying weights for children [weights of 0.2, 0.3, and 0.5 are given to children aged 0 to 6, 7 to 12, and 13 to 17, respectively]. Another equivalence scale which has been widely used gives a weight of 0.5 to all children. This can be written as: $AE = A + 0.5C$, where AE, A, and C have the same interpretation as before. The last two equivalence scales take into account only the differences in household composition. To date the question of which adult equivalence scale is appropriate remains an unresolved issue. One can easily percieve other equivalence scales which incoporates the gender dimension.

Having made the necessary adjustments for household size and composition, welfare comparisons are made by applying the dominance method to the adjusted expenditure data. In brief, dominance analyses involve Lorenz comparisons, first order dominance, and, where appropriate, it also considers second order dominance. In addition, income mobility and equity issues are addressed by analyzing mobility matrices using panel data.

Poverty comparisons are based on the Sen (1976) poverty index and its components, the head count ratio, the poverty gap ratio, and the Gini coefficient among the poor. Ravallion (1993) provides a summary of the dominance analysis as it applies to poverty comparisons. This approach relies on poverty value curves, i.e. the line joining poverty indices at various poverty lines, and tests for poverty value curve dominance across entire distributions. This has implications for poverty lines that may be drawn at any point within the distribution. Inference procedures of the sort used by Bishop, Formby, and Zeager (1995) and Zheng, Cushing, and Chow (1995) are used to evaluate poverty across time.

4. Measurement of Welfare, Inequality, and Poverty

4.1 The Dominance Method of Analysing Income Distributions

The dominance method is quite general and proceeds by ranking entire distributions in a series of steps or stages, which, in the early literature, are referred to as "degrees of stochastic dominance." More recently, these degrees of stochastic dominance have been referred to as simply first, second, and higher orders of dominance (c.f. Ravallion, 1994). The dominance method has its foundations in the literature of financial economics where it is used to evaluate utility maximizing choices of risky investment portfolios corresponding to alternative distributions of financial returns. The dominance method is used to derive a partial ordering of different portfolios of investments that are always preferred in the sense that they yield higher expected utility.

Atkinson (1970) had the basic insight that extended the dominance approach to income distributions and applied welfare economics. Rather than maximizing an individual decision maker's utility function, Atkinson and the analysts who followed in his path sought to maximize a general social welfare function. In this approach the income distribution is the counterpart of the distribution of investment returns in financial analysis. The dominance methodology as it is applied to welfare economics is well documented in the literature. To that extent, only a brief review is given in this paper.

4.2 First Degree Stochastic Dominance or First Order Dominance (FSD or FOD)

Quirk and Saposnik (1962) proved the original theorems on first order dominance, and we begin with a brief review of these results. The dominance methodology aims at ranking entire income distributions using an objective criteria. The criteria should be such that it reflects the ethical judgement of society and is generally acceptable to individuals of widely varying backgrounds. To arrive at the desired criteria the first order dominance (FOD) approach starts by making the following assumptions about the social welfare function (SWF).

1. The SWF is additively separable in individuals' utility functions;
   i.e. $W(X) = U_1(x_1) + U_2(x_2) + \ldots\ldots + U_n(x_n)$, where $W(X)$ is the SWF and $U_i(x_i)$ is the utility function for individual i, and x is income.

2. The individuals' utility functions are non-decreasing in income, x, i.e., $\frac{\partial U(x)}{\partial x} \geq 0$. Given assumption (1) above this also implies that the SWF is non-decreasing in income i.e., $\frac{\partial W(X)}{\partial X} \geq 0$. This assumption implies that society prefers more income to less.

3. The SWF is invariant to replication. Mathematically this implies that $W(\Pi X) = W(X)$, where Π is any scaling factor.

4. The SWF is symmetric, i.e., the SWF is unaffected if any two individuals swap places in the income distribution. This is sometimes referred to as the anonymity assumption.

FOD makes only one restriction on the form the SWF may acquire, i.e. that the first derivative of the SWF with respect to income is non-negative. This implies that the SWF can be concave, convex, or even concave in some portions of the distribution and convex in others, as long as it is non-decreasing in income. Figure 1 below shows three possible forms the SWF of this sort may take. In the finance literature, A would be termed risk neutral, B as risk loving, and C as risk averse. In welfare economics SWF A is said to show no preferences on inequality, SWF B shows a preference for inequality, and SWF C is equality preferring. The implication of this is that FOD would not have any information regarding inequality since the SWF can take any of the three forms or any combination thereof.

To arrive at the FOD criteria we consider two income distributions F and G whose probability density functions are f(x) and g(x), respectively, where x is an income vector arranged in ascending order. We further let F(x) and G(x) be the cumulative probability density functions of the two distributions. The SWF would show a preference for distribution F over distribution G if $W_F(x) \geq W_G(x)$, i.e., if welfare from distribution F is no less than that from distribution G. As shown in Appendix A, the FOD criteria that satisfies the above condition is that $F(x) \leq G(x)$ for all x, with strict inequality holding for some x. This condition is referred to as weak FOD. From a statistical point of view weak FOD requires that the difference between G(x) and F(x) be insignificantly different from zero at some points in the distribution, and statistically significant at one or more other points.

Figure 2 below illustrates the situation in which distribution F dominates distribution G in the sense that incomes in distribution F are everywhere higher than those in distribution G. Whether or not distribution F weakly or strongly dominates distribution G is a statistical question that is addressed in later sections of this paper.

A stronger version of FOD requires that distribution F be preferred to G if $W_F(x) > W_G(x)$, i.e., if the welfare from distribution F is larger than that from distribution G. The condition for strong FOD is that $F(x) < G(x)$ for all x.

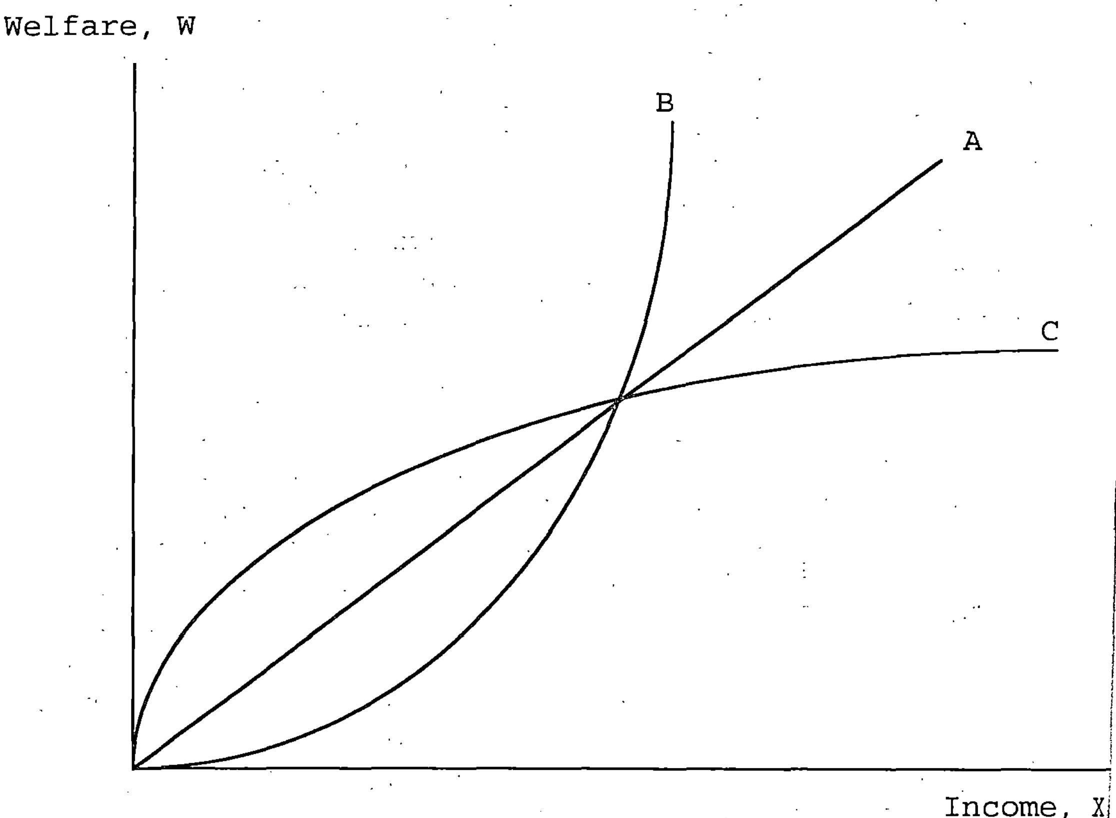


FIGURE 1: POSSIBLE FORMS OF THE SOCIAL WELFARE FUNCTION.

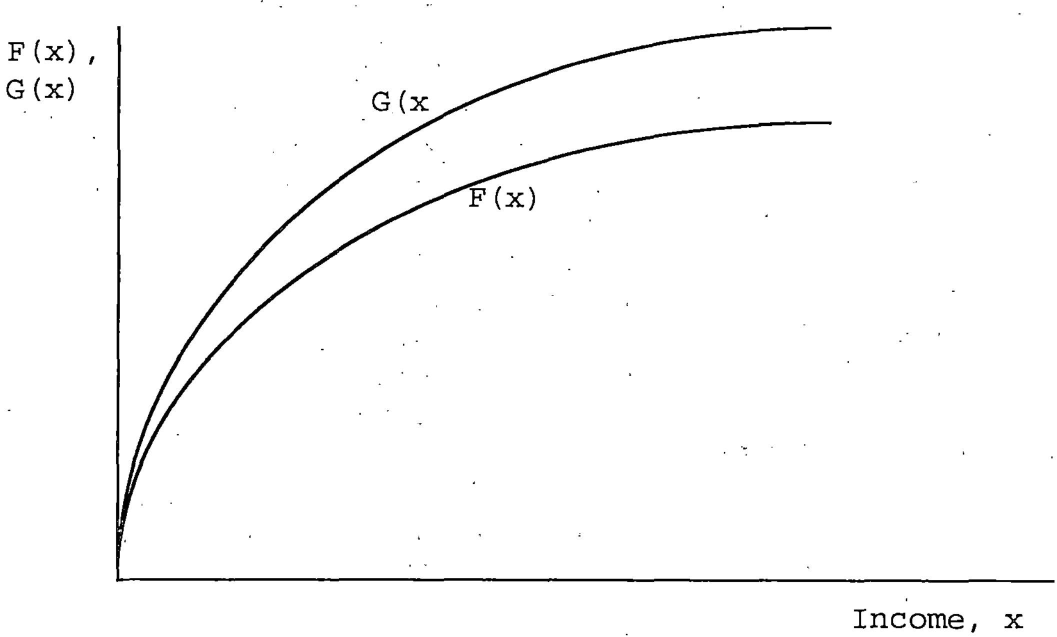


FIGURE 2. AN ILLUSTRATION OF THE DOMINANCE OF DISTRIBUTION F OVER DISTRIBUTION G.

One difficulty with FOD is that it is not likely to rank all distributions of interest, i.e. application of FOD results in only a partial ordering of distribution functions. In the event that FOD fails to rank the distributions of interest, higher order dominance, obtained by making further restrictions on the forms the SWF may take can be applied. In applied welfare economics the practice has been to end at the second order dominance (SOD), which involves assuming the Pigou-Dalton principle of transfers. The Pigou-Dalton principle of transfers asserts that a transfer of income from a high income person to a low income person, both of whom are below the poverty line, would increase welfare. Appendix A provides the mathematical derivation of both the FOD and the SOD conditions.

4.3 Rank Dominance and FOD

Saposnik (1981, 1983) demonstrates that rank dominance is equivalent to FOD and strong Pareto dominance. This equivalence is established under the assumptions of Rawls's veil of ignorance and symmetry. The Rawls's veil of ignorance assumes that we have a group of individuals who are to choose between two income distributions but they do not know their ultimate positions in those distributions. Symmetry implies that if two persons exchange positions in the income distribution, then the level of welfare remains unchanged.

Distribution F is said to rank dominance distribution G if $F(x_j) = G(x_i)$ and $x_j \geq x_i$ for all i and j with strict inequality holding for some $i \neq j$. The equivalence of rank dominance and FOD is demonstrated by deriving the rank dominance conditions using Figure 3 below. Inspection of the diagram reveals that for any value of x, say at x_2, $c = G(x_2) \geq F(x_2) = b$, which is the condition for FOD. Using the same diagram, which we know satisfies FOD conditions, we see that for $c = F(x_3) = G(x_2)$ $x_3 \geq x_2$, which is the condition for rank dominance.

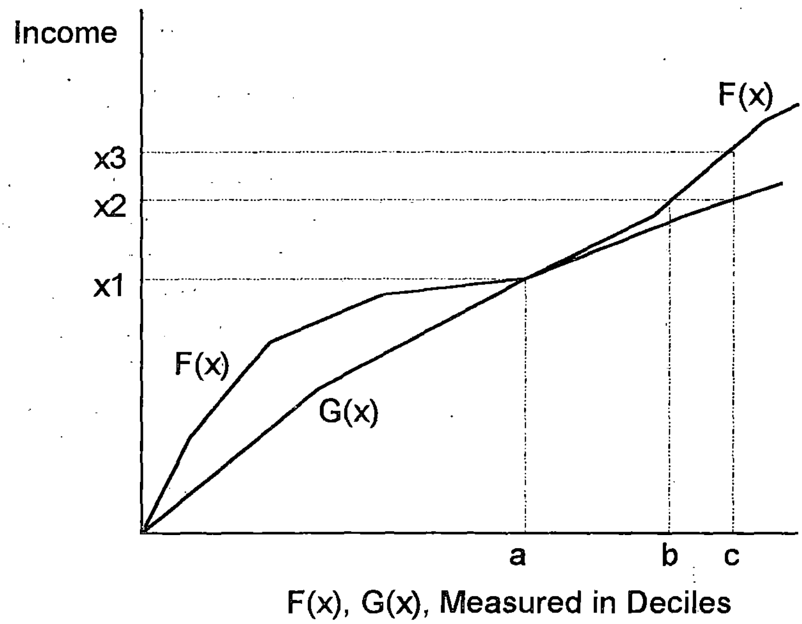


FIGURE 3: AN ILLUSTRATION OF THE EQUALITY OF FOD AND RANK DOMINANCE

The above stated condition for rank dominance is referred to as weak rank dominance. Strong rank dominance conditions require that $F(x_j) = G(x_i)$ iff $x_j > x_i$ for all i and j.

4.4 Lorenz Dominance

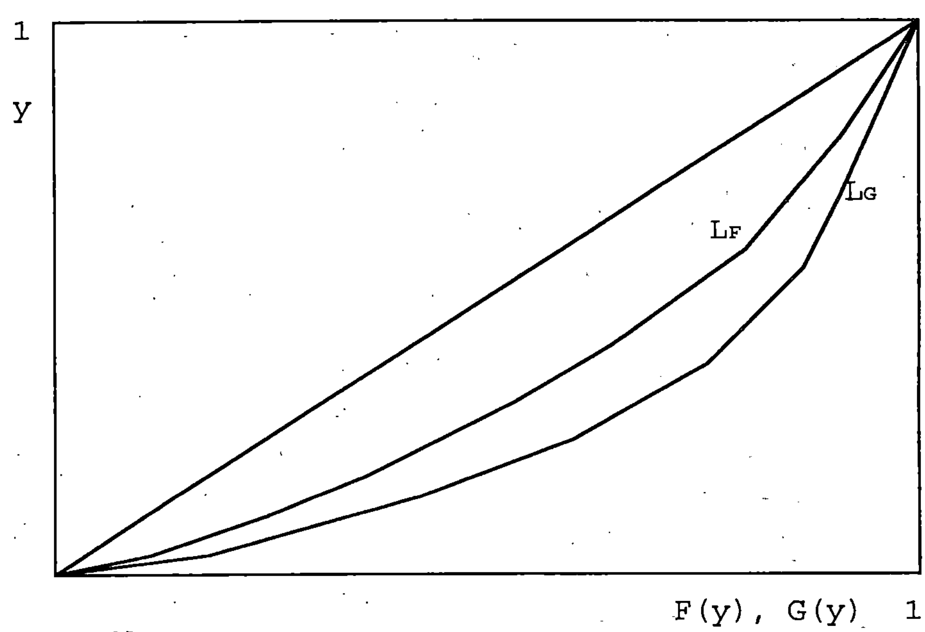
Atkinson (1970) and a number of influential writers including Shorrocks (1983) and Kakwani (1993) approached the dominance method in applied welfare economics by using the concept of a Lorenz curve. As is well known, a Lorenz curve shows the proportion of total income received by designated bottom percentiles of the population. Following Gastwirth (1971), the Lorenz curve can be written as $L_y(p) = \mu^{-1}\int_0^b yF^{-1}(y)dy$. We know that $F^{-1}(y) = f(y)$, and hence $L_y(p) = \mu^{-1}\int_0^b yf(y)dy$, or $\mu L_y(p) = \int_0^b yf(y)dy$. The Lorenz curve can therefore be defined implicitly by $\phi(F) = \frac{1}{\mu}\int_0^b yf(y)dy$, where $F = \int_0^b yf(y)dy$, and μ is the mean of the distribution. Integrating $\phi(F)$ by parts gives $\mu\phi[F(y_b)] = y_b F(y) - \int_0^b F(y)dy$. If we have two distributions, F and

G, then their Lorenz curves can be written as

$\mu_F\phi[F(y_b)] = y_bF(y) - \int_0^b F(y)dy$, and $\mu_G\theta[G(y_c)] = y_cG(y_c) - \int_0^c G(y)dy$.

Comparing the two Lorenz curves at $F(y_b) = G(y_c) = \bar{F}$ we have $\mu_F\phi(\bar{F}) - \mu_G\theta(\bar{F}) = [y_b - y_c]\bar{F} + [G_1(y_c) - F_1(y_b)]$, where $F_1(.) = \int F(y)dy$ and $G_1(.) = \int G(y)dy$. If we assume that $\mu_G = \mu_F = \mu$ then we have $\mu[\phi(\bar{F}) - \theta(\bar{F})] = [y_b - y_c]\bar{F} + [G_1(y_c) - F_1(y_b)]$. For distribution F to Lorenz dominate distribution G we require

$\mu[\phi(\bar{F}) - \theta(\bar{F})] = [y_b - y_c]\bar{F} + [G_1(y_c) - F_1(y_b)] \geq 0$. This would be the case if $y_b \geq y_c$, and $G_1(y_c) \geq F_1(y_b)$. As shown in Appendix A, this is the same condition necessary for SSD. Lorenz dominance of distribution F(y) over distribution G(y) is illustrated in Figure 4 below.



Notes:
1. F(y) and G(y) are the cummulative proportion of total number of households.
2. y is the cummulative proportion of total household incomes.
3. L_G and L_F are the Lorenz curves for distributions G and F, respectively.

FIGURE 4. AN ILLUSTRATION OF A CASE WHERE DISTRIBUTION F LORENZ DOMINATES DISTRIBUTION G.

It was through this mechanism that Atkinson (1970) connected Lorenz dominance to stochastic dominance and welfare. Dasgupta, Sen and Starrett (1973) generalized Atkinson's welfare results by showing that they apply to all social welfare functions of the S-concave class.

For distributions with the same mean, Lorenz dominance is equivalent to SSD. Atkinson (1970) points out that if the means are not the same, then for the above condition to hold it is necessary that the mean of distribution F to be no less than that of G. Sen (1973), however, pointed out that when the means are not equal then the Lorenz principle is devoid of welfare content. Nevertheless, even when the means are unequal Lorenz dominance remains the most general measure of income inequality. Thus, since inequality is of interest in its own right Lorenz dominance is an important part of the dominance methodology. Shorrocks (1983) extends the Lorenz method by demonstrating that even when the means are different the distributions can be rescaled by their means and Lorenz dominance comparisons can still be applied. Kakwani (1994) develops a very similar analysis. This approach is referred to as the generalized Lorenz (GL) dominance. In almost all cases in empirical studies the income distributions of interest have different means, and therefore GL or, equivalently, SOD comparisons would apply.

5. The Measurement of Poverty

The dominance method yields ordinal rankings of distributions of welfare, and most discussions of poverty seek a cardinal measure of deprivation in an economy. Only this type of measure can reveal the extent and depth of poverty. To devise an appropriate cardinal measure requires that we identify the poor (the identification problem) and that we be able to measure poverty in all of its relevant dimensions (the measurement problem). These problems are discussed below.

5.1 The Measurement Problem

To measure poverty we need a bench-mark poverty line, below which a person is considered poor. The choice of a poverty line is in the realm of normative economics hence it tends to be arbitrarily chosen. The problem of choosing a poverty line is only one of many measurement issues that have to be addressed in poverty research. Here I discuss those that are most relevant to the study of poverty in a poor country. The major problems include the following:

(a) the choice of an appropriate measure of household well-being;

(b) adjustment of the appropriate measure of well-being for household sizes and composition; and

(c) the choice of which poverty index to use.

Below I explore these problems further and explain how they are dealt with in this paper.

(a) The Most Appropriate Measure of Household Well-being

A central issue is whether household income or household consumption is a better measure of welfare. Compelling arguments for consumption expenditures as the preferred metric have been advanced (Ravallion, 1994). Further, the World Bank's design methodology of the original LSMS surveys were predicated on the idea of obtaining reliable measures of consumption expenditures as well as home based (in kind) consumption that does not flow through the market.

(b) The Problem of Differences in Household Size and Composition

Differences in household size and composition influence welfare. A large body of literature establishes that these differences can be taken into account by dividing the consumption data by an adult equivalence scale. The equivalence scale problem has already been discussed in an earlier section of this paper, and hence no further discussion is necessary at this point.

(c) The Poverty Index Number Problem

The poverty index number problem refers to the problem associated with having to chose which poverty index to use in poverty comparisons among the many competeting indices.

Beginning with the work of Sen (1976), it has been widely recognized that a good poverty measure should satisfy certain appealing axioms. According to Sen, an index of poverty must be distribution sensitive, which means that a transfer of income among the low income population must be reflected in the overall measure of poverty. In particular, if income is redistributed from an extremely poor person to a higher income person below the poverty line, the measure of poverty should increase, not decrease. There are a large number of poverty indices that satisfy this sensitivity criterion including the Watts (1968) index; the distinct classes of indices associated with Kakwani (1980); Clark, Hemming, and Ulph (1981); Foster, Greer and Thorbecke (FGT) (1984); and the pioneering index of Sen (1976). Surveys of the immense literature relating to these and other poverty indices are provided by Foster (1984), Seidl (1988), and Zheng (1994).

The choice among the competing poverty indices is in part dependent upon the purpose of the investigation and the type of analysis to be conducted. For example, some of the indices are additively decomposable, e.g., the FGT class, whereas others, including the Sen index, are not. In most poverty studies we are concerned with overall poverty and not additive decompositions. The general practice is to use the Sen index and its components; the headcount poverty ratio, H; the ratio of the average income shortfall-to-the poverty line, I, (i.e. the poverty gap ratio); and the Gini coefficient of income inequality among the poor, G_p, for poverty comparisons. The Sen index, which is generally denoted as S, can be written as

$$S = H[I + (1 - I)G_p \frac{q}{q+1}]$$

where H, I, and G_p are defined above, and q is the number of people below the poverty threshold. This approach is preferred because we are able to capture both the extent and the intensity of poverty. The headcount ratio and the poverty gap ratio provide a cardinal dimension to the poverty comparisons, while the Gini coefficient among the poor add an ordinal flavour. The Sen index combines both the cardinal and the ordinal aspects to the poverty index.

5.2 The Identification Problem

The identification problem is not trivial. The usual approach is to choose a poverty line below which individuals can be considered poor. But the choice of a poverty line involves value judgment and is essentially arbitrary. Bishop, Formby, and Smith (1993) and Bishop, Formby, and

Zeager (1995) applied inference dominance procedures to poverty comparisons. The poverty dominance criteria can be stated as follows: Distribution F poverty dominates (PD) distribution G iff for all poverty indices P, $P_G(Z_i) \geq P_F(Z_i)$ for all i, with strict inequality holding for some i, where the subscripts G and F on P refer to the distributions from which the poverty index is calculated, and z is the poverty line. We refer to this as weak poverty dominance. Strong poverty dominance of distribution F over distribution G requires that for all poverty indices, P, $P_G(Z_i) > P_F(Z_i)$ for all i. The poverty dominance comparison is said to be inconclusive if $P_G(Z_i) > P_F(Z_i)$ for some i and $P_G(Z_j) < P_F(Z_j)$ for some $j \neq i$. Note that if we say distribution F poverty dominates distribution G we mean that there is more poverty in distribution G than in F.

Foster and Shorocks (1988) show that for all SWF that satisfy the monotonicity conditin, i.e. $\frac{\partial W(X)}{\partial X} \geq 0$, poverty dominance implies FOD. This provides us with a vehicle through which we can link the poverty dominance comparisons with FOD comparisons.

Figure 5 below illustrates the conditions for poverty dominance.

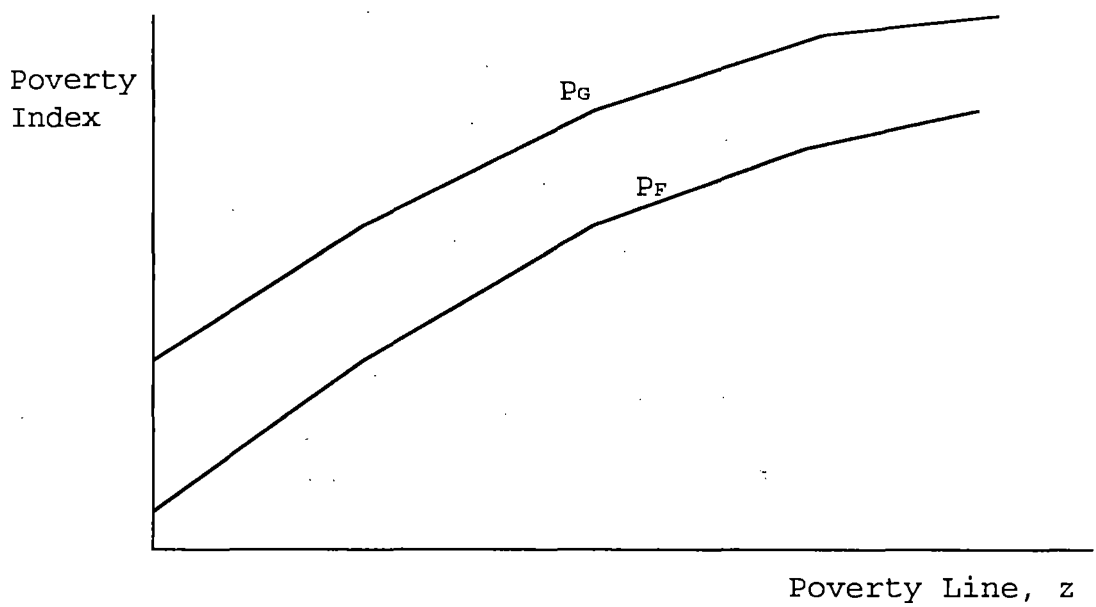


FIGURE 5. AN ILLUSTRATION OF POVERTY VALUE CURVE DOMINANCE OF DISTRIBUTION F OVER DISTRIBUTION G.

In the illustration the poverty value curve P_G (i.e., the line joining all the poverty indices at the various poverty lines) is persistantly above the poverty value curve P_F. Poverty dominance requires that this be the case for all poverty measures.

6. Statistical Inference Procedures

6.1 Introduction

Statistical test procedures are used to make inferences about differences in the ordinates of the distributions being compared. Specifically, we test for differences in Lorenz curves, quantile functions, and differences in poverty indices. The test procedures for testing differences in Lorenz curves and Quantile funtions, when the samples being compared are independent, are based on test procedures developed by Beach and Davidson (1983) and Beach and Richmond (1985). Beach and Davidson (1983) showed that differences between Lorenz ordinates are asymptotically normally distributed. They develop a covariance-variance (Ω) structure for these differences that can be consistently estimated without making prior assumptions about the underlying distribution. Using this covariance-variance structure they construct a Z-statistic that is asymptotically distribution-free and standard normal that can be used to make statistical inferences concerning the difference between corresponding Lorenz ordinates from the two distributions being compared. The calculated Z-statistics are compared with their critical values from the Student Maximum Modulus (SMM) table.

The Beach and Davidson inference procedure requires that the samples being compared be independent. In our case this implies that the data sets being compared should not contain panel households. Bishop, Chow, and Formby (1994) adapted the original Beach and Davidson procedures and developed similar procedures that can be used in studies involving matched samples that are dependent. Bishop, Formby, and Zheng (1994) show that the Sen poverty index and its three components, the Gini coefficient among the poor, the poverty gap measure, and the head count ratio, individually and jointly have asymptotically normal distributions. They extended the inference procedures to poverty comparisons. In performing joint tests they follow Beach and Richmond (1985) and test for differences in the poverty indices and compare calculated Z-statistics with their critical values in the SMM table.

The application of the inference procedures is fairly standard. In all the dominance comparisons (FOD, Lorenz dominance, and poverty dominance) we are drawing inferences from a union of disjointed subhypotheses, ten in the case of FOD, nine in the case of Lorenz curve analysis, and for poverty comparisons this number is determined by the number of pre-selected poverty lines. In the case of FOD we are making inferences about the welfare of the households in the two distributions being compared using sub-hypotheses concerning the differences of the ten mean decile household expenditures. In the case of Lorenz comparisons we are making inferences about income inequility in the two time periods being compared using subhypotheses concerning the differences of the nine Lorenz ordinates. We have nine points to compare in the case of Lorenz comparisons since by construction Lorenz curves start at 0 and end at 1, giving us nine points at which Lorenz ordinates can meaningfully be compared. Poverty dominance, on the other hand, relies on sub-hypotheses about differences in the poverty indices at pre-selected poverty lines to make inferences concerning poverty in one period or region compared to that in another.

The interpretation of the dominance comparison results is essentially the same for all the domiance comparisons. There are four possible outcomes when testing for the dominance of one distribution over another. From the given information we can make the following possible conclusions:

1. There is not enough evidence to suggest that the two distributions being compared come from different populations. In this case we conclude that the two distributions being compared are statistically the same or equal, or, put differently, the differences between the ordinates being compared is statistically not different from zero.

2. There is evidence to suggest that the two distributions being compared are drawn from different populations. In this case we conclude that the two distributions being compared are statistically different, or equivalently the differences between the ordinates being compared are statistically different from zero. This would be an example of strong dominance of one distribution over another.

3. The evidence shows statistical equality as described in (1) above in some portions of the distribution and strong dominance as described in (2) above in others. In this case we conclude that one distribution weakly domiantes the other.

4. The evidence shows strong statistical dominace of one distribution over the other in one portion of the distribution before turning into statistical equality and eventually being strongly dominanted statistically by the other distribution. In this case the results are incoclusive. In the case of FOD, decisive results may be obtained by applying higher orders of domiance.

These four possible outcomes are illustrated below. Since the interpretations are essentially identical for all the domiance comparisons only one illustration is furnished for each form of interpretation. That is to say that the illustrations presented are applicable for FOD, Lorenz dominace, and poverty dominance. Before going into the illustrations I present below a statement of the problem in the form of null hypothesis testing.

We test the null hypothesis: H_0: $D_i=0$ against the alternative hypotheses H_1: $D_i>0$; H_2: $D_i=0$ for some i and $D_j>0$ for some $j \neq i$; and H_3: $D_i>0$ for some i, and $D_j<0$ for some $j \neq i$. D refers to the difference in the ordinates being compared. The subscripts i and j on D refer to the point at which the ordinates are being compared. Thus $i,j=1,2,...,m$, where m is the total number of points at which comparisons are being made. For FOD m=10, while for Lorenz curve analysis m=9, and for poverty comparisons the value of m depends on the number of pre-selected poverty lines at which poverty indices are being compared. Acceptance of the null hypothesis, H_0, is equivalent to arriving at conclusion 1 discussed above. Rejection of the null hypothesis and accepting the alternative hypothesis H_1 is equivalent to arriving at conclusion 2, while acceptance of alternative hypothesis H_2 is equivalent to arriving at conclusion 3, and acceptance of H_3 is equivalent to arriving at conclusion 4.

An alternative way of stating the above hypothesis testing problem relies on the fact that $D_i=Y_i-X_i$, where Y_i is the ordinate from one of the distribution and X_i is the corresponding ordinate from the other distribution being compared with. The problem can now be equivalently stated as follows: H_0: $Y_i=X_i$; H_1: $Y_i>X_i$; H_2: $Y_i=X_i$ for some i and $Y_j>X_j$ for some $j\neq i$; and H_3: $Y_i>X_i$ for some i and $Y_j<X_j$ for some $j\neq i$.

The two ways of stating the problem have implications on how the four possible conclusions stated above can be illustrated diagramatically. Below I illustrate the diagrammatic interpretations of these four possible results. I also show how the same conclusions can be arrived at by looking at the test statistics.

6.2 An Illustration of Statistical Equality Using the Second Formulation of the Null Hypothesis

In Figure 6 below I draw two graphs, one showing the various values of Y_i at various points of comparisons (e.g. deciles for FOD) and the other showing corresponding values of X_i. I construct confidence bands around the curves. At the 95% confidence level of confidence these bands are constructed by taking two standard errors on either side of the curve to give the lower bound and the upper bound for the values of Y_i and X_i. In this case the lower bound for Y_i is everywhere inside the confidence band for X_i. Confronted with a given value of either Y_i or X_i we are unable to say which distribution it came from, that for Y_i or that for X_i, at least with 95% confidence. We therefore conclude that the two distributions being compared are statistically equal, or equivalentntly the differences between the ordinates being compared are statistically not differnt from zero.

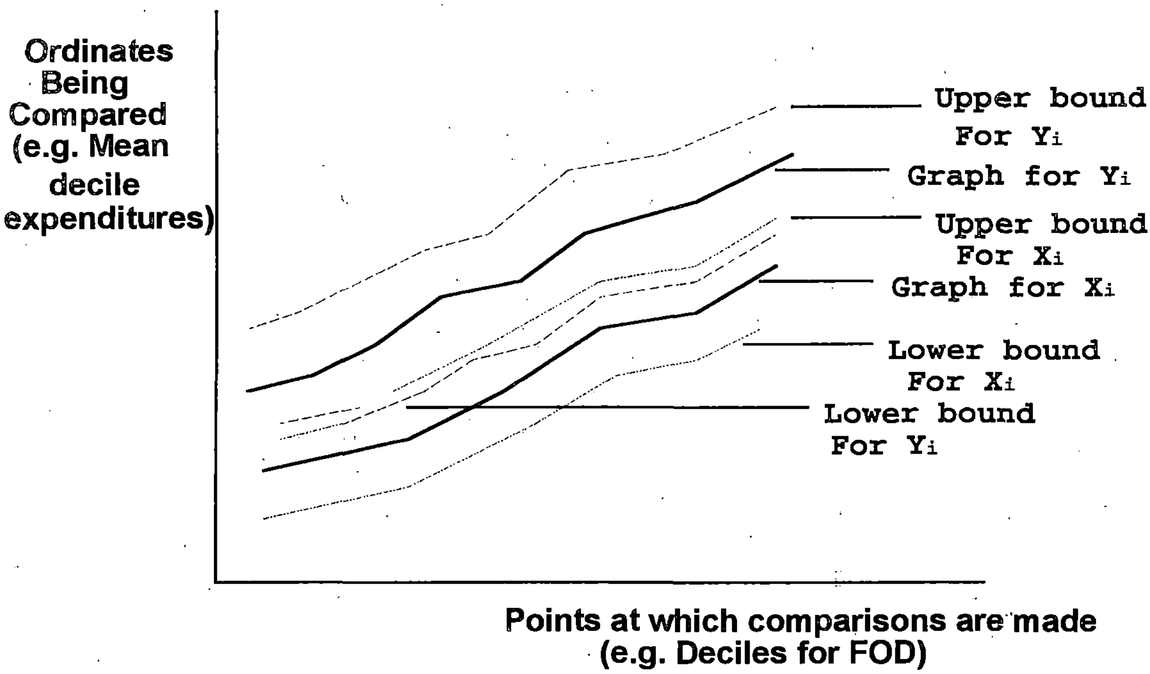


FIGURE 6: ILLUSTRATION OF STATISTICAL EQUALITY USING THE SECOND FORMULATION OF THE NULL HYPOTHESIS

6.3 An Illustration of Statistical Eqality Using the First Formulation of the Null Hpothesis

Figure 7 below illustrates statistical equality of two distributions being compared using the first formulation of the null hypothesis. Here we test the hypothesis that the difference between the ordinates being compared is equal to zero. We therefore construct the 95% confidence band around the horizontal line passing through the origin by taking two standard errors of the difference on either side of the line. Next we draw the graph of the differences in the ordinates being compared. If these differences all lie within the confidence band, as is the case in Figure 7, then we can say with 95% confidence that the two distributions being compareed are statistically equal, or equivalently the differnces between the ordinates being compared are not statistically different from zero.

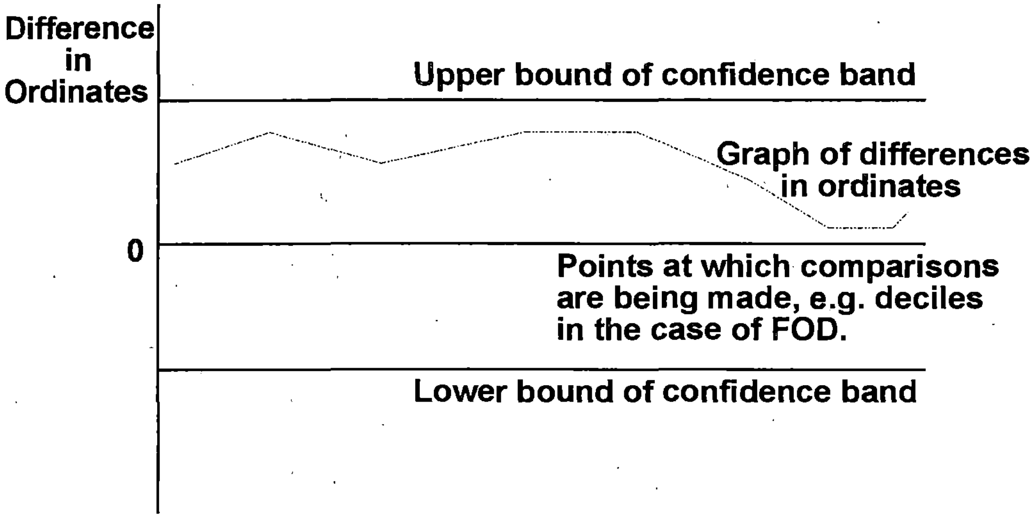


FIGURE 7: AN ILLUSTRATION OF STATISTICAL EQUALITY USING THE FIRST FORMULATION OF THE NULL HYPOTHESIS

6.4 An Illustration of statistical equality using the Analysis of the Test Statistics

Statistical equality can also be established by analyzing the test statistics of the difference between the pair of ordinates being compared. Table 1 below shows the results of the FOD comparisons of household foodshares in the Peruvian study (See Kaliyati, 1995) between 1985/86 and 1991. Here the results are being used for illustration purposes. While the results are specific for the analysis carried out, the interpretation of the results is the same for other dominance comparisons. Whether or not the results confirm conclusion 1, 2, 3, or 4 depends on whether or not the calculated values of the test statistics at the points being compared are equal to, less than, or greater than the critical values. With specific reference to Table 1 the critical value of the test statistic is 2.56. An inspection of the calculated values reveals that they all are less than the critical value. We therefore conclude that the difference between the ordinates being compared is statistically not different from zero. Put differently, the distributions from which the compared ordinates were drawn are statistically equal.

Table 1

First Order Dominance Comparisons For The Rural Highlands Region in Peru 1985/86 and 1991 (Household Food Shares)

Decile	Quantile Ordinate 1985/86 (%)	Quantile Ordinate 1991 (%)	Difference in Quantile Ordinates	Z - Statistic
1	95 (0.2202)	96 (0.3284)	-1	-0.25
2	91 (0.2129)	91 (0.3070)	0	0.00
3	87 (0.2110)	89 (0.3004)	-2	-0.54
4	85 (0.2144)	86 (0.3090)	-1	-0.27
5	81 (0.2275)	83 (0.3266)	-2	-0.50
6	77 (0.2418)	80 (0.3536)	-3	-0.70
7	73 (0.2570)	77 (0.3859)	-4	-0.86
8	67 (0.2857)	72 (0.4383)	-5	-0.96
9	58 (0.3262)	64 (0.4969)	-6	-1.01
10	39 (0.4144)	45 (0.6122)	-6	-0.81

Note: 1. The numbers in parenthesis are the variances

6.5 An Illustration of Strong Statistical Dominance Using the Second Formulation of the Null Hpotheis

Figure 8 below illustrates how strong statistical dominance of one distribution over another is established using the second formulation of the null hypothesis. Figure 8 is very similar to Figure 6. The only major differnce is that the two confidence bands do not cross. When this is the case we conclude that the distribution from which Y_i was drawn statistically dominates that from which X_i was drawn.

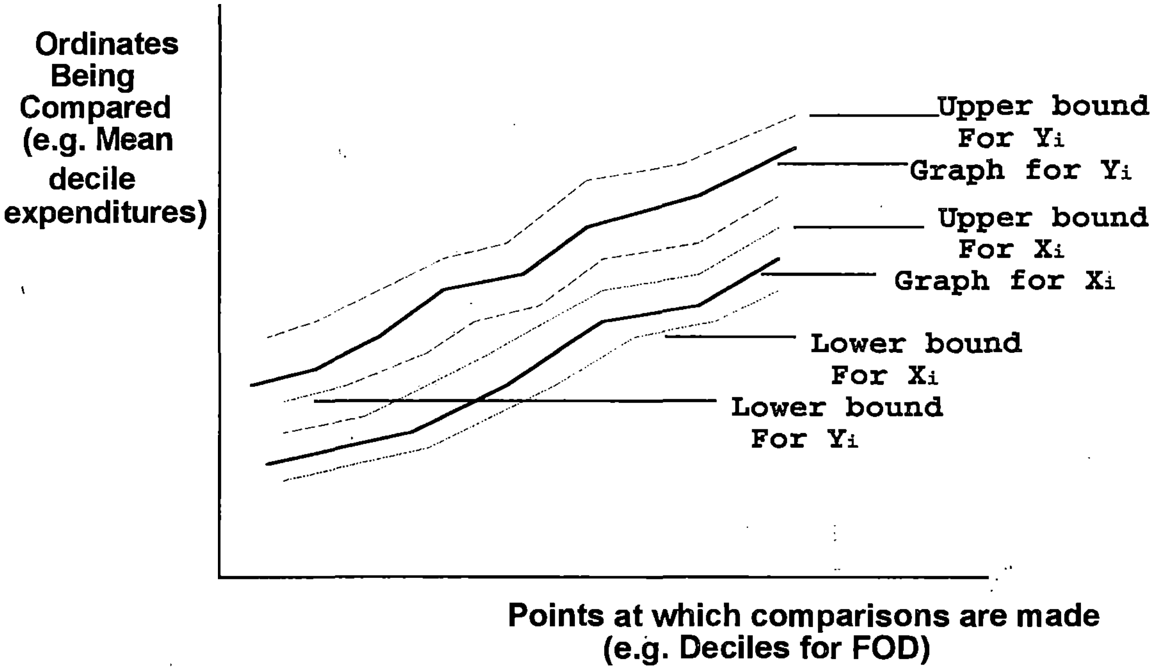


FIGURE 8: AN ILLUSTRATION OF STRONG STATISTICAL DOMINANCE USING THE SECOND FORMULATION OF THE NULL HYPOTHESIS

6.6 An Illustration of Strong Statistical Dominacne Using the First Formulation of the Null Hypothesis

Figure 9 below illustrates how strong statical dominace of one distribution over another is established using the first formulation of the null hyupothesis. Figure 9 is very similar to Figure 7. The only major difference is that the graph of the differences in the ordinates being compared lie outside the confidence band. When this is the case we conclude that the distribution from which Y_i was drawn statistically dominates that from which X_i was drawn, or equivalently the differences between the ordinates being compared are statistically differnt from zero.

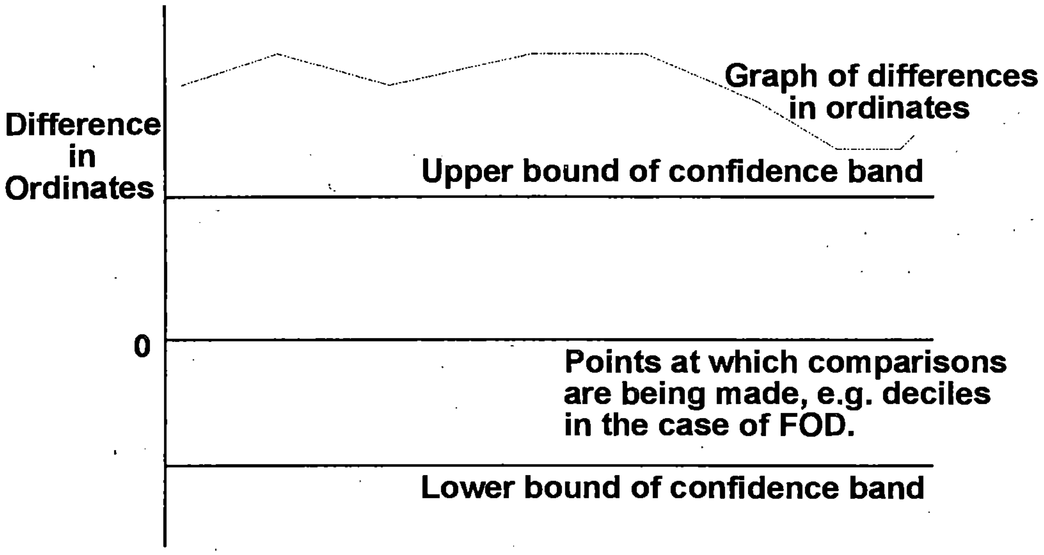


FIGURE 9: AN ILLUSTRATION OF STRONG STATISTICAL DOMINANCE USING THE FIRST FORMULATION OF THE NULL HYPOTHESIS

6.7 An Illustration of Strong Statistical Domiance Using the Analysis of the Test Statistics

Table 2 below follows exactly the same format as that for Table 1. Table 2 shows the results of Lorenz dominance comparisons in the Peruvian Study (Kaliyati, 1995) between 1990 and 1991 using total household expenditure per adult equivalence (AE2) as a welfare indicator. Again, the results are being used solely for the purpose of illustrating how strong statistical dominance is established using the test satistic. While the results are specific for the analysis carried out the interpretation of the results is the same for other dominance comparisons. Whether or not the results confirm conclusion 1, 2, 3, or 4 depends on whether or not the calculated test statistics at the points being compared are equal to, less than, or greater than the critical value. With specific reference to Table 2, the critical value of the test statistics is 2.63 at the 95% confidence level, and all the calculated values of the test-statistics are greater than the critical value. When this is the case we conclude that the distribution from which Y_i was drawn statistically dominates that from which X_i was drawn. Put differently, the differences between the ordinates being compared are statistically different from zero.

Table 2

An Illustration of Strong Statistical Dominance Using the Results from Lorenz Dominance Comparisons for Peru, 1990 to 1991 Using Total Expenditure Per Adult Equivalence (AE2)*

Decile	Lorenz Ordinate 1990	Lorenz Ordinate 1991 (%)	Difference in Lorenz Ordinates	Z-Statistic
1	2.48 (0.0007)	3.32 (0.0007)	-0.84	-8.44
2	6.45 (0.0015)	8.44 (0.0013)	-1.99	-9.91
3	11.50 (0.0024)	14.58 (0.0019)	-3.08	-9.99
4	17.33 (0.0035)	21.52 (0.0025)	-4.19	-9.85
5	24.01 (0.0046)	29,39 (0.0031)	-5.38	-9.72
6	31.77 (0.0058)	38.18 (0.0037)	-6.41	-9.27
7	41.00 (0.0072)	48.40 (0.0043)	-7.40	-8.78
8	52.46 (0.0088)	60.64 (0.0049)	-8.18	-8.13
9	67.58 (0.0104)	75.53 (0.0055)	-7.95	-6.76
10	100 (0.0000)	100 (0.0000)	0.00	0.00

* AE2=Adults+0.2*Kids06+0.3*Kids712+0.5*Kids1317 (Glewwe Type)

Notes:

1. The numbers in parenthesis are the standard errors.
2. The critical value of the Z-statistic at the 95% confidence level is 2.63.

6.8 An Illustration of Weak Statistical Dominance Using the Second Formulation of the Null Hypothesis

Figure 10 below is constructed in a similar way to Figures 6 and 8. In this case the lower bound of the confidence band for Y_i is within the confidence band for X_i between ordinates O_1 and O_2. Between these ordinates we cannot distinguish whether a given value of Y_i or X_i was drawn from the distribution from which Y_i or X_i was drawn. In other words the two distributions being compared are statistically equal between these two ordinates and different elsewhere. When this is the case then one of the distributions being compared is said to be weakly dominate by the other. In this instance the graphs of Y_i and X_i do not cross; however, weak dominance can also occur when the graphs actually cross. The dominance is termed weak if the lower bound of the confidence band of Y_i does not cross the lower bound of the confidence band for X_i. A case where this happens is also termed a weak crossing or an apparent crossing.

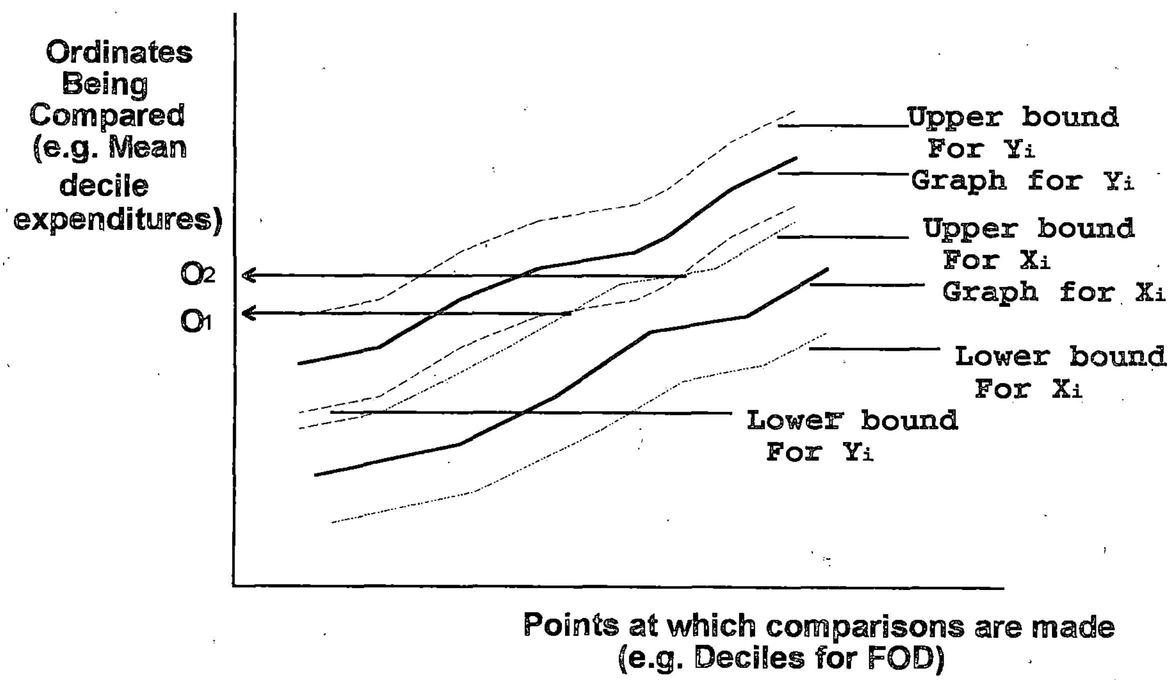


FIGURE 10: AN ILLUSTRATION OF WEAK STATISTICAL DOMINANCE USING THE SECOND FORMULATION OF THE NULL HYPOTHESIS.

6.9 <u>An Illustration of Weak Statistical Dominance Using the First Formulation of the Null Hypothesis</u>

Figure 11 below is constructed in a similar way to Figures 7 and 9. In this case the graph of the difference between the two ordinates being compared crosses the upper bound of the confidence band at A and B. Between these two points the differences between the ordinates being compared are not statistically different from zero. Between these two points we cannot distinguish whether a given value of Yi or Xi was drawn from the distribution from which Yi was drawn or that from which Xi was drawn. In other words the two distributions being compared are equal between these two points and different elsewhere. When this is the case then one of the distributions being compared is said to weakly dominate the other. In this instance the graph of the difference in ordinates does not cross the horizontal line passing through the origin. However, even if the graph of the difference crosses the horizontal line through the origin the dominance would still be termed weak if the graph of the differences does not cross the lower bound of the confidence band. A case like this one would be reffered to as a weak crossing or an apparent crossing.

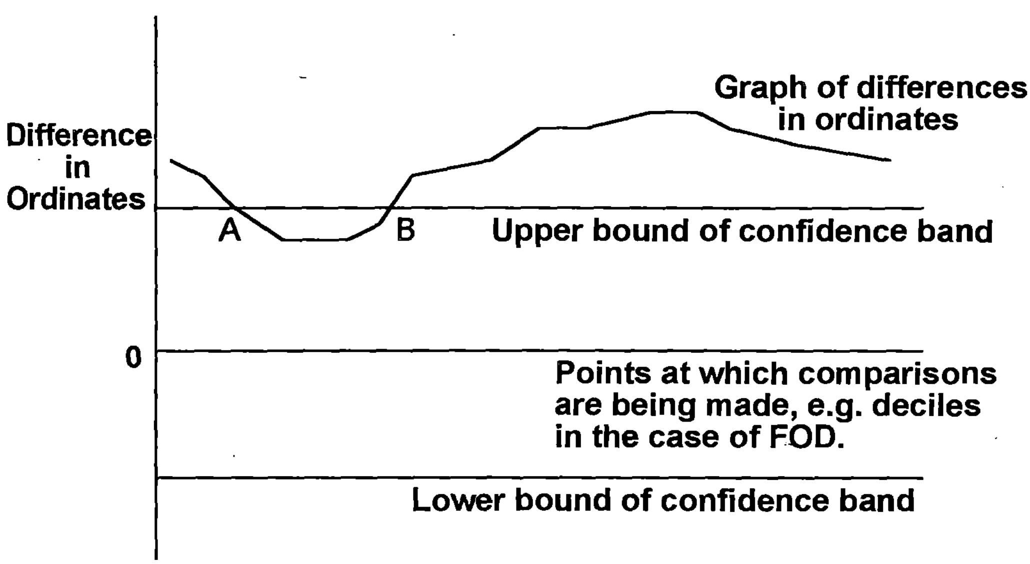


FIGURE 11: AN ILLUSTRATION OF WEAK STATISTICAL DOMINANCE USING THE FIRST FORMULATION OF THE NULL HYPOTHESIS

6.10 An Illustration of Weak Statistical Dominance Using the Analysis of the Test Statistics

Table 3 below follows exactly the same format as that for Tables 1 and 2. Table 3 shows the results of Lorenz dominance comparisons for Peru between 1985/86 and 1990 using total household expenditure per adult equivalence (AE3) as a welfare indicator. The results are being used solely for the purpose of illustrating how weak statistical dominance is established using the test statistic. While the results are specific for the analysis carried out, the interpretation of the results is the same for other dominance comparisons. Table 3 shows that the calculated values of the test statistics are greater than the 95% confidence level critical value of 2.63 at the first, second, third, and fourth deciles, while they are smaller at deciles 5 to 9. When this is the case we conclude that the distribution from which Yi was drawn statistically weakly dominates that from which Xi was drawn. Put differently, the differences between the ordinates being compared are weakly statistically different from zero. In this instance the differences between the ordinates does not change signs from positive to negative. However, even if the differences were to change signs we could still have weak dominance if the test statistic remains below the critical value after the change of sign.

6.11 An Illustration of an Inconclusive Result Using the Second Formulation of the Null Hypothesis

This occurs when the two confidence bands actually cross. This is illustrated in Figure 12 below. In the case of FOD conclusive results may be obtained by applying higher orders of dominance.

Table 3

An Illustration of Weak Statisticl Dominance Using the Results from Lorenz Dominance Comparisons for Peru,1985/86 to 1990 Using Total Expenditure Per AdultEquivalence (AE3)*

Decile	Lorenz Ordinates 1985/86 (%)	Lorenz Ordinates 1990 (%)	Difference in Lorenz Ordinates	Z-Statistic
1	2.91 (0.0005)	2.41 (0.0008)	0.50	5.24
2	6.97 (0.0011)	6.20 (0.0017)	0.77	3.88
3	11.90 (0.0017)	10.91 (0.0027)	0.99	3.09
4	17.62 (0.0024)	16.41 (0.0039)	1.21	2.64
5	24.40 (0.0032)	22.79 (0.0052)	1.61	2.62
6	32.35 (0.0041)	30.35 (0.0068)	2.00	2.53
7	41.74 (0.0051)	39.27 (0.0085)	2.47	2.50
8	53.10 (0.0061)	50.64 (0.104)	2.46	2.04
9	68.17 (0.0071)	65.77 (0.0126)	2.40	1.67
10	100 (0.0000)	100 (0.0000)	0.00	0.00

* AE3=Adults+0.5*Kids

Notes: 1. The numbers in parenthesis are the standard errors.
2. The critical value of the Z-statistic at the 95% confidence level is 2.63.

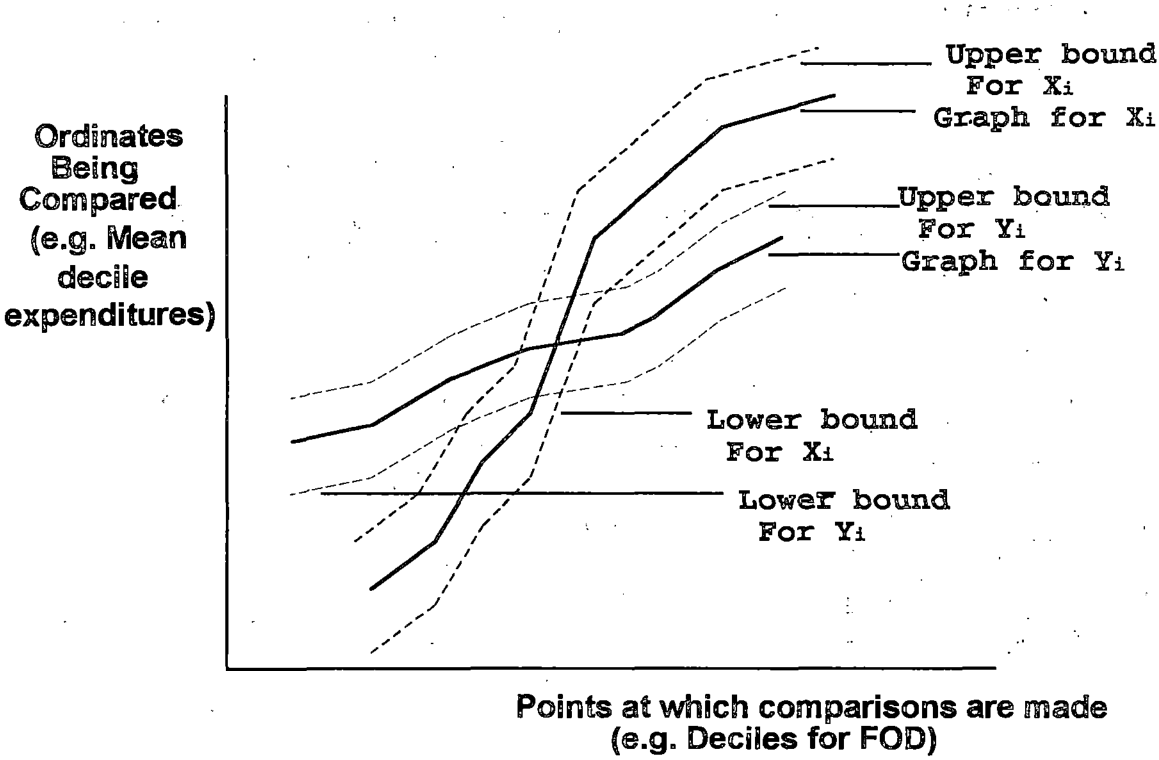


FIGURE 12: AN ILLUSTRATION OF AN INCONCLUSIVE DOMINANCE RESULT USING THE SECOND FORMULATION OF THE PROBLEM

6.12 An Illustration of an Inconclusive Result Using the First Formulation of the Null Hypothesis

This occurs when the graph of the differences in the ordinates being compared is at some points of comparison above the confidence band before passings through the confidence band and crossing on to the other side of the confidence band. This is illustrated in Figure 13 below.

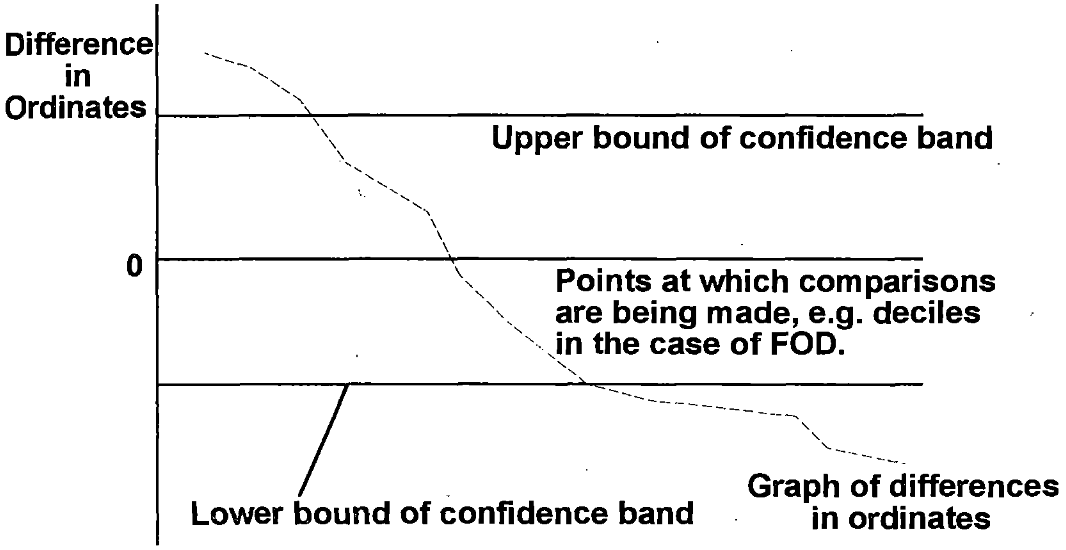


FIGURE 13: AN ILLUSTRATION OF AN INCONCLUSIVE DOMINANCE RESULT USING THE FIRST FORMULATION OF THE NULL HYPOTHESIS

6.13 An Illustration of an Inconclusive Result Using the Analysis of the Test Statistics

This conclusion is arrived at when the difference between the two ordinates being compared changes signs and at least one of the test statistics is positive and significant and at least one test statistic is negative and significant. This case is illustrated in Table 4 below.

Table 4

An Illustration of an Inconclusive Result Using Per Capita Total Expenditure Based Lorenz Dominance Comparisons for the Rural and Urban Areas in Highlands Region of Peru for 1991

Decile	Lorenz Ordinate Rural (%)	Lorenz Ordinate Urban (%)	Difference in Lorenz Ordinates	Z-Statistic
1	2.73 (0.0005)	2.44 (0.0007)	0.29	3.36
2	6.85 (0.0011)	6.71 (0.0014)	0.14	0.79
3	12.09 (0.0018)	12.20 (0.0021)	-0.11	-0.40
4	18.69 (0.0026)	18.98 (0.0027)	-0.29	-0.78
5	26.40 (0.0032)	26.91 (0.0032)	-0.51	-1.13
6	35.31 (0.0038)	36.20 (0.0038)	-0.89	-1.65
7	45.78 (0.0044)	47.29 (0.0043)	-1.51	-2.45
8	58.25 (0.0049)	60.31 (0.0047)	-2.06	-3.04
9	73.84 (0.0052)	76.19 (0.0049)	-2.35	-3.31
10	100 (0.0000)	100 (0.0000)	0.00	0.00

Notes:
1. The numbers in parenthesis are the standard errors.
4. The critical value of the Z-Statistic at the 95% confidence level is 2.63.

6.14 Conclusion

The dominance methodology is a powerful analytical tool that is used in applied welfare economics to infer the general direction of welfare across time. In the absence of further analysis aimed at identifying who is becoming better off and who is getting worse off, the stochastic dominance methodology would present an incomplete picture of welfare changes. At the policy-making level it is important that we know whether or not the poor remained poor across time. This knowledge would complete the picture of how welfare was changing across time. In the following section I present the mobility matrix analyses which show the extent to which individuals or households were changing their welfare positions in society across time.

7. **Income Mobility Analysis**

7.1 **Introduction**

Welfare mobility shows the degree to which individuals or households are changing their welfare ranking in society across time. Two related types of welfare mobility analyses are available, namely, transition or mobility matrix analysis and the Lorenz-concentration curve analysis. In this section I provide a brief overview of mobility matrix analysis. The Lorenz-concentration curve analysis will be dealt with in another paper.

7.2 **Mobility Matrix Analysis**

Atkinson, Bourguignon, and Morrison (1988) offer three reasons why income mobility analysis is an essential complement to the conventional cross-sectional studies. The first reason is that it is essential in policy-making as it relates to social programs. The second reason relates to the importance of characterizing the poor across time. The third is technical in the sense that inequality is deemed to be a short-term phenomenon with inequality vanishing in the long-run because of mobility.

Interest in income mobility has ranged from attempts at measuring mobility (Paris, 1955; Dardanoni, 1993; Atkinson, Bourguignon, and Morrison, 1988; Geweke, Marshall, and Zarkin, 1986; Conlisk, 1990; and Markandya, 1984) to applying it in measuring the impact of income tax evasion on wlfare (Formby et al., 1995). Shorrocks (1978) links income inequality and income mobility.

This paper recognizes the imprtance of income mobility analysis, particularly in developing countries where absolute and relative poverty allevation is at the centre of most, if not all, policy issues. It deserves emphasis that income mobility analysis is a dissertation topic in its own right. Income mobility analysis in this paper does not go into the theoretical foundations of this kind of analysis. Rather, the income mobility analysis is used to suppliment the dominance analysis. As such it is approached only from a descriptive standpoint. A description of mobility matrices and their interpretation is therefore given below.

Mobility matrices trace the movement of a given individual or household within an income distribution across time. They are therefore constructed using panel data sets. An illustration of how the mobility matrix is constructed is done using the 727 panel households in the 1985/86 and 1990 Peruvian LSLSs. The procedure involves the following steps:

1. Arrange the 1985/86 households into deciles by adjusted total household expenditures.

2. Arrange the 1990 households into deciles by adjusted total household expenditures.

3. Merge the 1985/86 and 1990 data sets by household identificataion number and sort the merged data set by 1985/86 deciles.

4. The mobility matrix is then constructed as an array of numbers showing the number of households occupying the new 1990 deciles for each of the 1985/86 deciles.

Table 5 shows the mobility matrix constructed in this way. The rows show the 1985/86 deciles, while the columns show the 1990 deciles. If we look along a row, say row 1, then the row total, 72, is the total number of households that were in decile 1 in 1985/86. The first number, 24, is the number of households that were in decile 1 in 1985/86 and remain in decile 1 in 1990. The second number in row 1, column 2, 17, is the number of households that were in decile 1 in 1985/86 and are now in decile 2 in 1990. The third number, 8, is the number of households that were in decile 1 in 1985/86 and are now in decile 3 in 1990, and so on. From the mobility perspective the 24 households that were in decile 1 in both time periods represents households that did not move or change their relative welfare positions between the two time periods. Given that there were a total of 72 households in decile 1 in 1985/86 it would follow that 48 households (Equal to the sum of the off-diagonal elements in row 1) moved or changed relative welfare positions between the two time periods. In percentages this implies that 67% of the households moved out of decile 1 between the two periods. 67% can therefore be used as a measure of mobility for decile 1 between 1985/86 and 1990. The same interpretation is used for the other rows.

Table 5

Mobillity Matrix for Peru for the Period 1985/86-1990 Using Per Capita Household Consumption Expenditure

	Decile 1990										
Decile 1985	1	2	3	4	5	6	7	8	9	10	Total
1	24	17	8	6	7	5	2	2	1	0	72
2	18	14	10	9	7	5	4	4	1	1	73
3	12	13	8	5	11	7	5	6	4	2	73
4	11	7	11	8	12	6	3	6	4	4	72
5	3	4	9	12	7	19	8	7	2	2	73
6	2	5	12	10	6	10	12	8	3	5	73
7	1	5	4	10	8	6	14	6	11	7	72
8	1	5	3	4	7	5	8	14	17	9	73
9	0	2	2	2	7	6	8	12	17	17	73
10	0	1	6	6	1	4	8	8	13	26	73
Total	72	73	73	72	73	73	72	73	73	73	727

We note, however, that households that were in decile 1 in 1985/86 could only move to higher deciles in 1990. On the other hand the households that were in decile 10 in 1985/86 could only move to lower deciles in 1990. The households in the intervening deciles could move in either direction with those at higher deciles having more room to move down than up and vice-versa. We also know that if we were to assign a + for increase in decile position and a - for a decline in decile position, then for the entire matrix the pluses would cancel out the negatives; i.e., it is a "zero-sum-game." Despite the "zero-sum-game" nature of mobility matrices, it is still possible to make inter-matrix mobility comparisons. The sum of the diagonal elements gives the total number of households in the total sample that did not change decile positions between the two time periods. The sum of the off-diagonal elements expressed as a percentage of the sample size is therefore an indicator of mobility for the entire mobility matrix. In this way it is possible to carry out inter-matrix mobility comparisons.

It is sometimes more convenient to present mobility matrices in terms of percentages of row totals. Table 6 below gives this type of representation of the mobility matrix shown in Table 5. In this case decile mobility indices would simply be 100 minus the diagonal element, and the mobility index for the entire matrix would be found by subtracting from 100 the sum of the diagonal elements divided by 100.

Preliminary results of the Peruvian study (Kaliyati, 1995) showed that the use of different equivalence scales to adjust for household sizes and compositions had negligible effect on the mobility matrix analysis results. Like in Lorenz dominance comparisons, mobility analysis is invariant to the method of deflating incomes across time. Therefore, hyper-inflation does not distort the mobility comparisons.

8. Welfare Comparison Methodologies and the Poverty Debate in Zimbabwe.

It was pointed out ealier that the debate on poverty in Zimbabwe has focused on:

(a) Poverty Measurement;
(b) Poverty Monitoring; and
(c) Poverty Alleviation/Eradication

To alleviate poverty or even eradicate it, requires the knowledge of who is poor and why he/she is poor. The debate on why a person is poor calls for a lot of value judgement. While this is the case we can at least enquire about the characteristics of the poor. To make this characterisation complete it is necessary to characterise the "non-poor".

Table 6

Mobillity Matrix for Peru for the period 1985/86-1990 Using Per Capita Household Consumption (% of Row Totals)

	Decile 1990										
Decile 1985	1	2	3	4	5	6	7	8	9	10	Total
1	33	24	11	8	10	7	3	3	1	0	100
2	25	19	14	12	10	7	6	6	1	1	100
3	16	18	11	7	15	10	7	8	6	3	100
4	15	10	15	11	17	8	4	8	7	6	100
5	4	6	12	16	10	26	11	10	3	3	100
6	3	7	16	14	8	14	16	11	4	7	100
7	1	7	6	14	11	8	19	8	15	10	100
8	1	7	4	6	10	7	11	19	23	12	100
9	0	3	3	3	10	8	11	16	23	23	100
10	0	1	8	8	1	6	11	11	18	36	100
Total	10	10	10	10	10	10	10	10	10	10	100

The importance of this lies in that one would want to know whether in fact the characteristics associated with the poor truly pertains to the poor. The question of who is poor is a simple issue that can easily be answered once people nanimously agree on the poverty line. In the absence of a poverty line that is unanimously agreed upon we can still use the dominance approach to answer the question, who is poorer than who by region, income sub-groups or any other desirable sub-divisions. In this section I look at the issue of poverty measurement and poverty monitoring in the context of the current debate on poverty in Zimbabwe.

From the methodological section it was made clear that when we are talking about poverty we are in fact talking about welfare. It was also pointed out that welfare comparisons comprise income inequality comparisons, poverty comparisons, income mobility analysis, and overall welfare comparisons. A more equitable distribution of income is prefered to a less equitable distribution. A reduction in poverty indices across time is preferable to an increase. Higher income mobility is prefered to less mobility in that it reflects vibrancy within the economy. Needless to say that a higher welfare is prefered to less. This methodology is not criticism proof. It can therefore be enriched by also the old ways of comparing welfare. Thus one can use school enrolment, infant mortality rates, stunting, wasting etc, to cross-check the results obtained from the dominance comparisons. This new methodology does not only suggest the type of data that needs to be available for poverty measurements and monitoring but it also suggests a number of other things. For example it suggests that the data should be collected on a regular basis and that part of the data set ought to be panel. Needless to point out that this type of analyses require the original micro data sets and not summary tables. It also suggests that poverty monitoring activities cannot be disjointly done by several organisations, unless somebody wants to cross-check the results obtained by another researcher.

Appendix A

The Mathematics of First and Second Degree Stochastic Dominance

The first degree and second degree stochastic dominance criteria are based on the following calculus. If we have two distributions F and G with probability density functions, pdf, f(x) and g(x), respectively, where x is a random number, which in our case is income, then the expected utility from the two distributions are given by*

$E_F U(x) = \int_a^b U(x) f(x) dx$; and (1)

$E_G U(x) = \int_a^b U(x) g(x) dx$ (2)

If we let $F(x) = \int_a^b f(x) dx$; and (3)

$G(x) = \int_a^b g(x) dx$ (4)

then F(x) and G(x) are the areas under the curves f(x) and g(x) between the intervals a and b, or alternatively they are the respective cumulative probability density functions, cpdf, of distributions F and G. From equations (3) and (4) it would follow that

$f(x) = F'(x)$, and (5)

$g(x) = G'(x)$. (6)

Substituting (5) and (6) into equations (1) and (2), respectively, gives

$E_F U(x) = \int_a^b U(x) F'(x) dx$, and (7)

$E_G U(x) = \int_a^b U(x) G'(x) dx$ (8)

Equations (7) and (8) can now be integrated using the integration by parts which says that

$\int w(x) v'(x) dx = w(x) v(x) - \int v(x) w'(x) dx$.

Letting $U(x) = w(x)$ and $F'(x) = v'(x)$ in equation (7), and $U(x) = w(x)$ and $G'(x) = v'(x)$ in equation (8), then equations (7) and (8) can be rewritten as

$E_F U(x) = U(x) F(x) \Big|_a^b - \int_a^b F(x) U'(x) dx$ (9)

$E_G U(x) = U(x) G(x) \Big|_a^b - \int_a^b G(x) U'(x) dx$ (10)

Now letting $F_1(x) = \int F(x) dx$, and (11)

$G_1(x) = \int G(x) dx$. (12)

* In this exposition utility function and social welfare functions are taken to be synonymous.

$F_1(x)$ and $G_1(x)$ are therefore the areas under the cpdf F(x) and G(x). From (11) and (12) it follows that

$$F(x) = F_1'(x)\text{, and} \tag{13}$$

$$G(x) = G_1'(x)\,. \tag{14}$$

$\int_a^b F(x)U'(x)dx$ in equation (9), and $\int_a^b G(x)U'(x)dx$ in equation (10) can now be rewritten as

$$\int_a^b F(x)U'(x)dx = \int_a^b U'(x)F_1'(x)dx\text{, and} \tag{15}$$

$$\int_a^b G(x)U'(x)dx = \int_a^b U'(x)G_1'(x)dx\,. \tag{16}$$

Using the integration by parts rule again, and letting $U'(x) = w(x)$ and $F_1'(x) = v'(x)$ in (15), and $U'(x) = w(x)$ and $G_1'(x) = v'(x)$ in (16), then (15) and (16) can be rewritten as

$$\int_a^b F(x)U'(x)dx = U'(x)F_1(x)\Big|_a^b - \int_a^b F_1(x)U''(x)dx\text{, and} \tag{17}$$

$$\int_a^b G(x)U'(x)dx = U'(x)G_1(x)\Big|_a^b - \int_a^b G_1(x)U''(x)dx\,. \tag{18}$$

Substituting (17) and (18) into equations (9) and (10) respectively gives

$$E_F U(x) = U(x)F(x)\Big|_a^b - U'(x)F_1(x)\Big|_a^b + \int_a^b F_1(x)U''(x)dx\text{, and} \tag{19}$$

$$E_G U(x) = U(x)G(x)\Big|_a^b - U'(x)G_1(x)\Big|_a^b + \int_a^b G_1(x)U''(x)dx\,. \tag{20}$$

First degree stochastic dominance, FSD, makes the assumption that $U'(x) \geq 0$. No restrictions are placed on higher order derivatives of the utility function hence no refutable hypotheses can come out of expressions of $E_F U(x)$ and $E_G U(x)$ involving higher order derivatives of the utility function. Thus, FSD makes use of equations (9) and (10). The FSD rule chooses distribution F over G iff $E_F U(x) \geq E_G U(x)$, or equivalently

$$\text{iff } E_F U(x) - E_G U(x) \geq 0\,. \tag{21}$$

Using equations (9) and (10) condition (21) implies that

$$U(x)[F(x) - G(x)]\Big|_a^b + \int_a^b [G(x) - F(x)]U'(x)dx \geq 0\,. \tag{22}$$

For a cpdf a=0, b=1, F(a)=G(a)=0, and F(b)=G(b)=1, which implies that F(b)-G(b)=0, and hence $U(x)[F(x) - G(x)]\Big|_a^b = 0$. Condition (22) can therefore be restated as

$$\int_a^b [G(x) - F(x)]U'(x)dx \geq 0\,. \tag{23}$$

Using the knowledge that an integral of a positive number is also positive we can write condition (23) as

$$G(x) - F(x)]U'(x) \geq 0\,. \tag{24}$$

The restriction that $U'(x) \geq 0$ implies that $G(x) - F(x) \geq 0$, or equivalently $G(x) \geq F(x)$, for condition (24) to hold. The FSD rule for distribution F to dominate G is that $E_F U(x) \geq E_G U(x)$ for all x with strict inequality holding for some x. This implies that $G(x) \geq F(x)$ for all x, with strict inequality holding for some x. Conversely, if we do not put any restriction on $U'(x)$ and simply require that $G(x) \geq F(x)$ for all x with strict inequality holding for some x, then condition (24) is satisfied iff $U'(x) \geq 0$ for all x, with strict inequality holding for some x.

Second degree stochastic dominance, SSD, makes two restrictions on the utility function, namely $U'(x) \geq 0$ and $U''(x) \leq 0$. No further restrictions are put on higher derivatives of the utility function, hence no refutable hypotheses can come out of the expressions of $E_F U(x)$ and $E_G U(x)$ involving derivatives of higher order than two. The SSD, therefore, makes use of equations (19) and (20). Distribution F is preferred to distribution G by SSD iff $E_F U(x) \geq E_G U(x)$, or equivalently iff

$$E_F U(x) - E_G U(x) \geq 0 \tag{25}$$

for all x, with strict inequality holding for some x. Using equations (19) and (20) condition (25) implies that

$$U(x)[F(x) - G(x)]\Big|_a^b + U'(x)[G_1(x) - F_1(x)]\Big|_a^b + \int_a^b [F_1(x) - G_1(x)]U''(x)dx \geq 0 . \tag{26}$$

Again, $U(x)[F(x) - G(x)]\Big|_a^b = 0$ (see section on FSD), and $G_1(a) = F_1(a) = 0$; hence condition (26) can be restated as

$$U'(b)[G_1(b) - F_1(b)] + \int_a^b [F_1(x) - G_1(x)]U''(x)dx \geq 0 . \tag{27}$$

Condition (27) would hold unambiguously if

$$U'(b)[G_1(b) - F_1(b)] \geq 0, \text{ and} \tag{28}$$

$$\int_a^b [F_1(x) - G_1(x)]U''(x)dx \geq 0 . \tag{29}$$

Under the assumption that $U'(x) \geq 0$ condition (28) is satisfied only if $G_1(b) \geq F_1(b)$. (30)

Under the assumption that $U''(x) \leq 0$ condition (29) is satisfied only if $G_1(x) \geq F_1(x)$. (31)

From equations (11) and (12) condition (31) also implies that $F(x) \leq G(x)$. SSD therefore incorporates all the conditions for FSD plus the proviso that $G_1(b) \geq F_1(b)$ and $G_1(x) \geq F_1(x)$. SSD therefore implies FSD, although the opposite is not always true. In other words SSD is a subset of FSD.

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